

ERGODIC THEOREMS AND GEOMETRY OF BANACH SPACES

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ABSTRACT Kakutani and Yosida proved (separately) that if a power-bounded operator T on a Banach space X , is (what we now call) weakly almost periodic (all orbits are weakly conditionally compact), then T is mean ergodic, and concluded that all power-bounded operators on a reflexive Banach space are mean ergodic.

In general, a power-bounded operator on a reflexive space need not be almost periodic (all the orbits are conditionally compact), even in Hilbert space; weakly mixing probability preserving transformations which are not mixing provide examples.

We analyze the ergodic properties of power-bounded operators on a reflexive Banach space, which have the form "scalar plus compact-power", and show that they are almost periodic. If such an operator is weakly mixing, then it is stable (its powers converge in the strong operator topology), and so is also its adjoint.

Let X_{ISP} be the separable reflexive indecomposable Banach space constructed by Argyros and Motakis, in which every operator has an invariant subspace. Every operator on X_{ISP} (and on its subspaces) is of the form $\alpha I + S$ with S^3 compact, and we conclude that every power-bounded operator on (a closed subspace of) X_{ISP} is almost periodic.

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