Measures on Cantor sets and their classification

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Two measures $\mu$ and $\nu$ on a Cantor set $X$ are called homeomorphic if there exists a homeomorphism $f$ of $X$ such that $\mu = \nu \circ f$. We are interested in the problem of classification of Borel probability and infinite measures on a Cantor set with respect to a homeomorphism. For a wide class of probability measures which E. Akin called good, a criterion of being homeomorphic is known. A full non-atomic measure $\mu$ is good if whenever $U, V$ are clopen sets with $\mu(U) < \mu(V)$, there exists a clopen subset $W \subset V$ such that $\mu(W) = \mu(U)$. For the class of good probability measures, the set $S(\mu)$ of values of measure $\mu$ on all clopen subsets of $X$ is a complete invariant.

We consider ergodic probability and infinite invariant measures for aperiodic substitution dynamical systems. S. Bezuglyi, J.Kwiatkowski, K.Medynets and B.Solomyak showed that these measures can be described as ergodic measures on non-simple stationary Bratteli diagrams invariant with respect to the cofinal (tail) equivalence relation. We also consider a wide class of infinite measures on a Cantor set and finite and infinite measures on a non-compact locally compact Cantor set. In every case, we find necessary and sufficient condition for good measures to be homeomorphic. It turns out, that for good infinite measures, the set $S(\mu)$ is not a complete invariant, we find a new invariant which is complete.