CENTRAL LIMIT THEOREM FOR $\mathbb{Z}^d$-ACTION
BY TORAL AUTOMORPHISMS

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Abstract

Let $\mathcal{S}$ be an abelian finitely generated semigroup of endomorphisms of a probability space $(\Omega, \mathcal{A}, \mu)$. If $T \in \mathcal{S}$, we write $T = T^{\hat{k}} = T_1^{k_1} \ldots T_d^{k_d}$, where $(T_1, \ldots, T_d)$ is a system of generators in $\mathcal{S}$ and $\hat{k} = (k_1, \ldots, k_d) \in (\mathbb{N}^+)^d$.

For $f \in L^2(\mu)$, limit theorems can be investigated for the random field $(f \circ T)_{T \in \mathcal{S}}$. For instance, given an increasing sequence of domains $(D_n) \subset (\mathbb{N}^+)^d$, a question is the convergence in distribution of the normalized sequence

$$|D_n|^{-\frac{1}{2}} \sum_{\hat{k} \in D_n} f \circ T^{\hat{k}}, \ f \in L^2_0(\mu).$$

After a preliminary spectral study when the action of $\mathcal{S}$ has a Lebesgue spectrum, we consider $\mathbb{N}^d$- or $\mathbb{Z}^d$-actions given by commuting toral endomorphisms on $\mathbb{T}^\rho$, $\rho \geq 1$.

Classical results on the existence and the construction of such actions by automorphisms will be briefly discussed. Then a CLT will be presented for (1) or related expressions, as well as a criterion of non-degeneracy of the variance, when $f$ belongs to a certain class of regularity on the torus.

Other examples of multi-dimensional actions of commuting endomorphisms, like the semigroup generated by $x \to p_j x \mod 1$ with $p_j \in \mathbb{N}^*$ on $\mathbb{T}^1$, are also discussed.

This is a joint work with Guy Cohen (Ben-Gurion University).

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